Relay synchronization in multiplex networks

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Synchronization is one of the most important collective phenomena in many natural, social and technological systems becoming a very active research topic in network science. The modeling of complex systems using graph theory has improved our understanding of the interplay between the topology of the arrangement of the interacting units and the emerging dynamics [1]. The huge amount of new data collected in the last years has permitted a higher resolution network representation of real systems. In particular, the inclusion of new features shaped multi-layer representations, i.e., approaches in which the network units are arranged in several layers, each one accounting for a different kind of interactions among the nodes [2]. Multi-layer structures determine scenarios where novel forms of synchronization are relevant, as unidirectional coordination between layers [3], intra-layer or inter-layer [4, 5] synchronization.

Very recently, relay and remote synchronization (two very well known phenomena in chains, or small motifs, of coupled oscillators) have captured the attention of researchers [6]. This form of synchronization is observed when two units of a network (identical or slightly different) synchronize despite not being directly linked, and due instead to the intermediation of a relay mismatched unit. Relay synchronization is of outstanding relevance in the brain: The thalamus acts as a relay between distant cortical areas through the thalamo-cortical pathways, playing the role of a coordination hub that maintains the information flow. Recently, remote synchronization has been addressed in the context of complex networks revealing the extremely important role of network structural and dynamical symmetries in the appearance of distant synchronization as it was already suggested by the observation of zero-lag delays between mirror areas of the brain. Nevertheless, the interplay between symmetry, dynamics and xsmulti-layer structure remains still unexplored.

In this talk, we report on the realization of relay synchronization in multiplex networks, where inter-layer synchronization occurs between distant layers mediated by a relay layer that acts as a transmitter (see Fig. 1). We show that this transmission can be extended to higher order relay configurations, provided symmetry conditions are preserved. By first order perturbative analysis, we identify the dynamical and topological dependencies of relay synchronization in a multiplex. We find that the relay synchronization threshold is considerably reduced in a multiplex configuration, and that such synchronous state is mostly supported by the lower degree nodes of the outer layers, while hubs can be demultiplexed without affecting overall coherence. Finally, we experimentally validated the analytical and numerical findings by means of a multiplex of three layers of electronic circuits.

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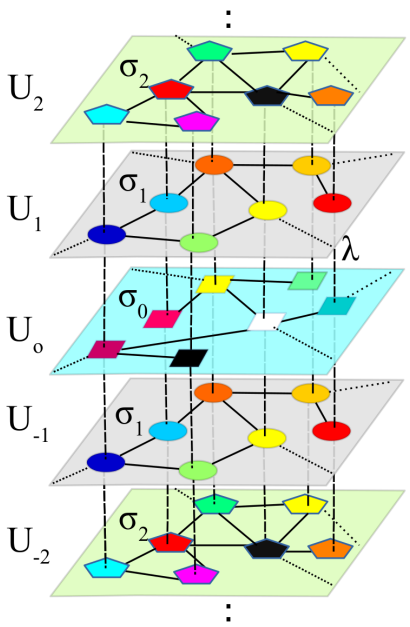

Fig. 1. Schematic representation of a multiplex of 5 layers, where each pair of layers k and $-k$ with $k = -2, -1, 0, 1, 2$, (painted with the same color) are networks of identical oscillators with the same topology and coupling strength and whose dynamical state is described by the variable $\mathbf{U}^k$ and $\mathbf{U}^{-k}$, respectively. The multiplex is symmetric with respect to the layer $k = 0$ that acts as the relay layer and the nodes are coupled to their replicas in the rest of layers with a different coupling strength.