Field theory for recurrent mobility

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Mobility flows are mathematically encoded using the so-called Origin-Destination (OD) matrices. Given a set of geographical areas, such matrices can be seen as weighted networks in which the nodes are the unit areas and the links point from the origin to the destination of the flow (see [1] for a recent review). Since these are recurrent mobility flows, these areas correspond to home and work locations, and besides the morning direction the flows also occurs on the opposite direction in the afternoon. These are therefore spatially embedded networks for which factors like the population density, job opportunities, location and distance play a fundamental role. Several models have been proposed in order to obtain the flows from these basic variables. The bet is high since determining transport demand is fundamental for infrastructure building and urban planning.

In this work, we introduce a new approach to the topic. It is based on the observation that the flows can be represented as vectors pointing from the origin to the destination, and that these elementary vectors can be summed to produce an average field in every unit area. This mechanism is illustrated for London in Fig. 1 with ODs coming from Twitter data, where each unit cell of 1×1 km^2 is depicted with its corresponding average vector. Furthermore, we found that this vector field fulfills the Gauss (divergence) theorem and also that its rotational is nearly zero in all the space. The first feature allows us to study the flux around different closed perimeters, we used essentially circles of different radius around the center of the cities. The classical models to reproduce OD matrices are then employed to generate fields and their results are tested against the empirical fields. The flux produced by a gravity model with an exponentially decaying deterrence function with the distance fits much better than the same model with other deterrence functions or the radiation model.

Additionally, the fact that the rotational is almost zero everywhere allows us to define a potential in the space reducing, thus, the dimensionality of the problem. The maximum of the potential is located in the center of the cities, and then it decays as one gets further. Interestingly, the extrema of the potential can be used to define different mobility attraction areas and to delimit the areas of influence of different cities as it can be seen in the case of the Manchester-Liverpool conurbation (Fig. 2). The results of this work will appear soon in a coming paper [2].

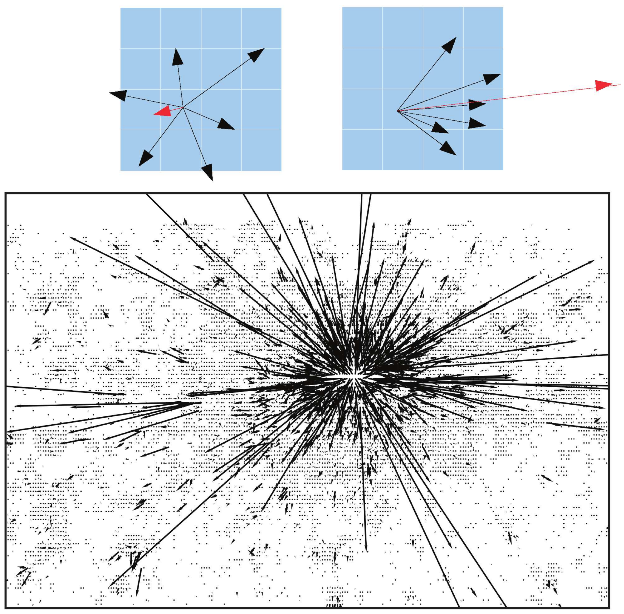

Fig. 1. Top row, two examples with the definition of the average vector in every cell (red vector). In the bottom, the vector field in an area comprehending the Greater London.

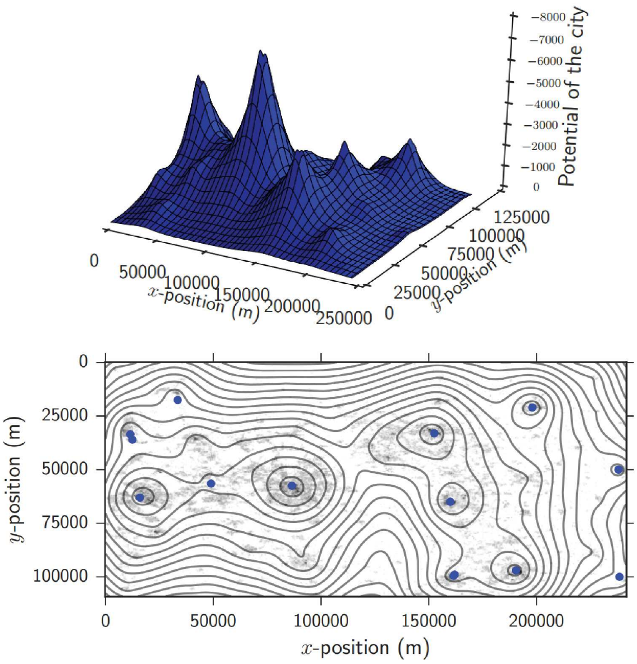

Fig. 2. The potential field calculated using the gravity model with an exponential deterrence function in the area of Manchester an Liverpool. We find 13 centers (local maxima).

[1] H. Barbosa, M. Barthelemy, G. Ghoshal, C. R. James, M. Lenormand, T. Louail, R. Menezes, J. J. Ramasco, and F. Simini, M. Tomasini, Human mobility: Models and applications, Phys. Rep. **734**, 1-74 (2018).

[2] M. Mazzoli, A. Molas, M. Lenormand, P. Colet, and J. J. Ramasco, Field Theory for recurrent mobility, (in preparation).