# Ageing-induced continuous phase transition

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Ageing is an ubiquitous effect in nature. It has different meanings, depending on the strand of research considered. Classical examples range from non-equilibrium statistical mechanics [1], where its effects are studied on spin glasses, to biology [2], considered as the increase of mortality with age of a species, to chemistry [3], where the properties of a material change over time without any external forces. In any case, ageing can be seen as the dependence of the dynamics of a system on an internal time, often heterogeneously distributed, of the individual components that form such a system. Specifically, we take the approach of considering ageing as the influence that persistence times, i.e., the time without changing state, have on the state transitions: it constrains the transitions in a way that the longer an element remains in a given state, the smaller is the probability to change it. The inclusion of this effect in the modelling part adds a realistic component into the description of a given problem and unveils new and rich phenomenology.

In this work [4] we investigate the effects of including heterogeneous time-dependent transitions on the critical properties of a stochastic model. We add ageing into the noisy voter model (also known as the Kirman model), a paradigmatic binary-state stochastic model appeared in several contexts: percolation, surface-catalytic reactions, probability theory, opinion dynamics, ... The agents modify their binary state by means of noise and pair-wise interactions. Interestingly, due to ageing the system passes from a finite-size discontinuous transition between ordered (ferromagnetic, bimodal) and disordered (paramagnetic, unimodal) phases to a second order phase transition, well-defined in the thermodynamic limit, belonging to the Ising universality class (see top and middle panels in Fig. 1). We characterize it analytically by finding the stationary solution of an infinite set of mean field equations. The theoretical predictions are tested with extensive numerical simulations in low dimensional lattices and complex networks. In addition, the ageing properties are employed to understand the symmetries broken in the phase transition (bottom panel in Fig. 1).

In summary, by adding a realistic ingredient in the modelling framework, we prove that ageing plays a central role in modifying the critical properties of a stochastic model. The studied phenomenology can occur in other complex systems beyond opinion dynamics models, with potential impact on a wide range of disciplines.

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[3] A. L. Robinson, N. M. Donahue, M. K. Shrivastava, E. A. Weitkamp, A. M. Sage, A. P. Grieshop, T. E. Lane, J. R. Pierce, and S. N. Pandis, Rethinking organic aerosols: Semivolatile emissions and photochemical aging, Science **315**, 1259-1262 (2007).

[4] O. Artime, A. F. Peralta, R. Toral, J. J. Ramasco, and M. San Miguel, Ageing-induced continuous phase transition, (under review).

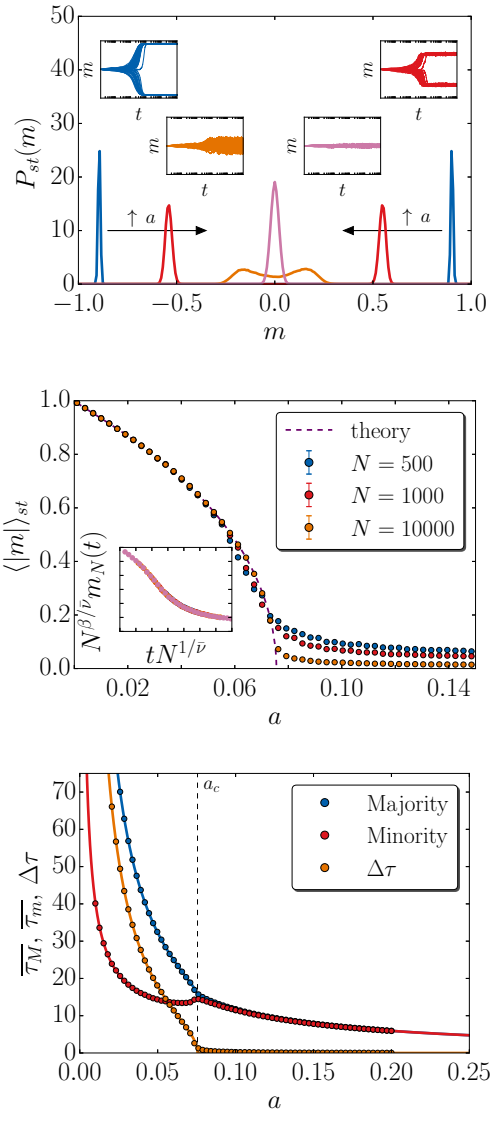


Fig. 1. (Top) Stationary probability distribution for the magnetization. Different curves correspond to different values of noise $a$. In the insets, the individual trajectories of the dynamics. (Middle) Phase diagram for the stationary value of the magnetization for different system sizes, together with the analytical curve. In the inset, collapse of the magnetization curves using the mean field Ising critical exponents. (Bottom) Mean internal times for the majority and minority population, and their difference, in function of the noise. Points are simulations and solid line is the analytical curve.