Cognitive resource allocation determines the organization of personal networks

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Some of the most robust findings about human social networks are concerned with the size and structure of the individuals' personal networks. These studies suggest that, among humans, an individual typically deals with about 150 relationships including kin and friends. These relationships are further organized into a set of hierarchically inclusive layers (*circles*) of increasing size with decreasing emotional intensity whose sizes follow a characteristic sequence with a scaling ratio close to 3 [1]: 5, 15, 50, 150. Although the overall size of the networks has been connected to our cognitive capacity [2], the layered structure and the consistent scaling ratio are experimental evidences for which no theoretical explanation has been given. By means of the maximum entropy principle [3], we show that the existence of a cost to relationships (in terms of time and/or cognitive investment) and heterogeneity in the relationships (in terms of their benefits and/or emotional content) naturally yield this outcome. Furthermore, we show that the fraction of links in circle k is, under certain conditions, given by

$$\chi_k = \frac{1 - e^{k\mu}}{1 - e^{r\mu}}, \qquad (1)$$

where r is the total number of circles and μ is the only parameter of the model.

The model not only accounts for the layered structure previously mentioned, but also predicts the existence of a new kind of regime. If $\mu > 0$, then the relationships are hierarchically distributed as it has been widely reported in the literature, with an approximately constant scaling ratio given by $x \approx e^\mu$ —we will call this the *standard* regime. However, if $\mu < 0$, then the individuals tend to have a large number of *close* relationships and little acquaintances —we call this the *inverse* regime. According to the model, the latter should be prompted when the number of available relationships for an individual is particularly small, or, more precisely, when the ratio between available resources and possible relationships is large. Importantly, this second type of structure had not been hitherto reported.

We analyze the standard regime using data from a group of 84 students from a major Middle Eastern university [4]. The results are summarized in Fig. 1 (a) and (b). Most individuals ($\sim 98\%$) have a value of $\mu > 0$, meaning that their circles show the standard structure, as expected. In order to test our prediction about the inverse regime, we focus on four different communities of immigrants whose sociological features suggest that they form independent, small social environments within their places of residence [5, 6]. Figure 1 (c) and (d) shows our results for one of these communities. Remarkably, 96% of its networks lie within the inverse regime with $\mu < 0$, confirming our hypothesis —the results are similar for the remaining three communities.
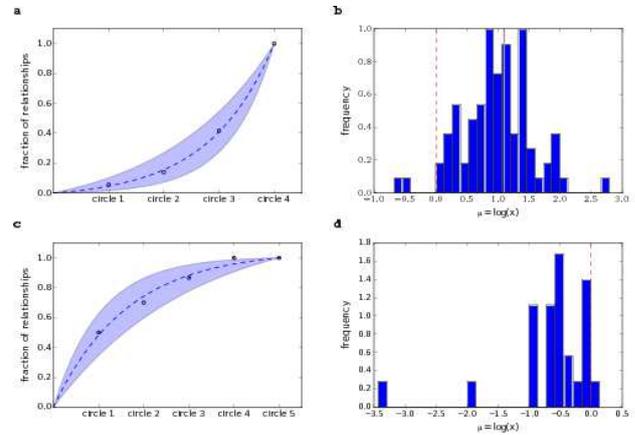

Fig. 1. Summary of the results of the data analysis. Upper panels summarize our results for the community of students, whereas lower panels summarize those for one of the communities of immigrants. Left panels show representative fittings for individuals in both communities. Solid dots represent experimental data, blue dashed lines represent the graph of the fitted model, and shaded regions show the 95% confidence interval for the parameter estimate. Right panels show the distribution of the parameter estimates (μ) for both communities. The red dashed lines mark the change of regime (i.e., $\mu = 0$). (a) Representative fitting for an individual in the community of students –standard regime. (b) Distribution of the parameter estimates for the community of students. The gray, solid line indicates the typically observed scaling ratio $x = 3$ ($\mu = 1.099$). (c) Representative fitting for an individual in one community of immigrants – inverse regime. (d) Distribution of the parameter estimates for one community of immigrants.

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