Dynamics and synchronization of complex networks with coupling delays and fluctuating topology

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The presence of delay in the interaction systems is an important problem in many real-life systems such as communication networks, neural systems or social networks. Of special interest is the study of networks with changing topology that might elucidate some important behaviors in different systems such as continuous changing neural networks due to the synaptic plasticity or real social networks. Other aspect of importance is the study of static hierarchically organized delay-coupled networks in order to understand how the hierarchy affect the functioning of these systems. In the present communication we study the generic properties of directed delay-coupled networks, which topology changes with time and make comments on the role of the hierarchical organization regarding synchronization properties.

In a previous publication [1] we numerically studied synchronization properties of delay-coupled networks with a time-varying topology. We considered an interaction network of coupled chaotic maps with a single coupling delay $T_{\rm d}$, with a topology fluctuating among an ensemble of Small-World networks, with a characteristic time-scale $T_{\rm n}$. We found that random network switching may enhance the stability of synchronized states, depending on the interplay between the time-scale of the delayed interactions $T_{\rm d}$ and that of the network fluctuations $T_{\rm n}$.

In this communication we consider an interaction network of coupled chaotic Bernoulli maps in a fluctuating topology directed small-world network with delay, where the third timescale is the internal time scale of the nodes $T_{\rm in}$. When the network fluctuations are faster than the coupling delay and the internal time scale ($T_{\rm n} \ll T_{\rm in}, T_{\rm d}$) the synchronized state can be stabilized by the fluctuations. As the network time scale $T_{\rm n}$ increases, the synchronized state becomes unstable when $T_{\rm n} \sim T_{\rm d}$. Synchronization is more probable as the network time scale increases further. However, in the slow network regime ($T_{\rm n} \gg T_{\rm d} \gg T_{\rm in}$) the long-term dynamics is desynchronized whenever the probability of reaching a non-synchronizing network is finite. In the intermediate regime the system shows a sensitive dependence on the ratio of time scales, and specific topologies, reproduced as well by numerical simulations.

These results have been complemented with analytical results in the linearized limit, where by using the Master Stability Function [2] on a network of alternating topology [3], we expressed the effective adjacency matrix in terms of the three time scales. We showed that when the network fluctuations are much faster than the internal time scale and the coupling delay ($T_{\rm n} \ll T_{\rm in}, T_{\rm d}$), the effective network topology is the arithmetic mean, while in the opposite case ($T_{\rm in} \ll T_{\rm n} = T_{\rm d}$), the effective topology is the geometric mean over the different topologies.

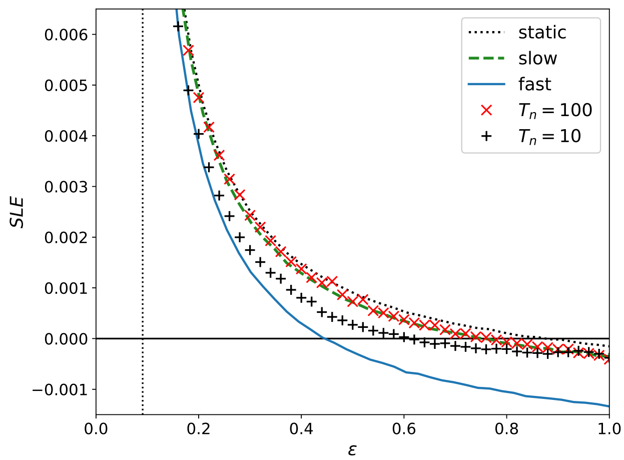

Fig. 1. Numerical synchronization Lyapunov exponent corresponding to the evolution of a time-varying delay-coupled Bernouilli network with $N = 40$ units and time-delay $\tau = 100$, see [3] for details. The solid blue and dashed green lines correspond to the fast and slow effective networks, respectively, obtained as the arithmetic and geometric ensemble mean matrices. We also plot the average SLE of the static case.

Future extensions of the research might be the study of other network ensembles, such as random Erdös-Rényi graphs, scale-free networks or even more complicated graphs of multiplex type with further application to real world problems concerning transport and energy issues or problems of supply networks in the general context of "smart cities".

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[3] O. D'Huys, J. Rodríguez-Laguna, M. Jiménez Martín, E. Korutcheva, and W. Kinzel, arXiv:1805.03004.