Networks competing between them

Javier M. Buldú[1,2,3], Jaime Iranzo[4], and Jacobo Aguirre[3,5]
[1]Laboratory of Biological Networks, Center for Biomedical Technology, UPM, 28223 Pozuelo de Alarcón, Madrid, Spain
[2]Complex Systems Group, Universidad Rey Juan Carlos, 28933 Móstoles, Madrid, Spain
[3]Grupo Interdisciplinar de Sistemas Complejos (GISC)
[4]National Center for Biotechnology Information, National Library of Medicine, National Institutes of Health, Bethesda, MD, USA
[5]Centro Nacional de Biotecnología, CSIC, c. Darwin 3, 28049 Madrid, Spain

Social, biological, physical and technological systems are composed of a diversity of interacting agents, leading network science, a statistical physics understanding of graph theory, to be a genuine tool for investigating their structure and dynamics [1]. Within the framework of social networks, the topology of the interactions between individuals has been demonstrated to be crucial in, for example, the vanishing of the critical threshold in epidemics [2] or the efficient and fast propagation of innovation [3]. In a similar fashion, the topology of a network itself can be influenced by the dynamical processes occurring in it, giving rise to adaptive mechanisms that rule the evolution of the structure of social networks [4].

The emergence of cooperation, defection or altruism can be investigated by linking game theory to network science. In this way, the intrinsic heterogeneity of social networks, the majority of them showing power-law distributions in the number of connections, has been related in many cases to the emergence of cooperation, contrary to what is observed in homogeneous populations [5]. Furthermore, highly connected individuals have also been shown to be more prone to collaborate than scarcely connected ones [6]. While attention was initially focused on the interplay between nodes' strategies and the structure of the underlying (single) network, more recently, coevolutionary rules have also been related to the emergence of interdependency [7] and multilayer structures [8]. But, what if we are concerned about the interests of a network as a whole instead of its nodes? Does it make sense to consider networks competing or collaborating with other networks? The fruitful recent literature about *networks-of-networks*, or in a more general context about *multilayer networks*, makes these two questions timely and extremely relevant [9]. A diversity of dynamical processes such as percolation [10], diffusion [11] or synchronization [12] have been recently reinterpreted by assuming that real networks unavoidably interact with other networks, a contact that may be beneficial or detrimental to each of the networks belonging to the ensemble.

Here we investigate how $m > 2$ networks compete or cooperate to achieve a relative increase of importance measured as eigenvector centrality, which maximizes their outcome in a variety of dynamical processes [14]. In our competition, networks can vary the way they interact with other networks, evolving in time until they reach a stable situation where all networks refuse to modify their strategy because any change would lead to a worse result. Importantly, an *a priori* optimal connection strategy for a given network may not be reachable due to the actions of the competitor networks, which turns the analysis of the final outcome of the networks into a study of Nash equilibria [13] in a network-of-networks. With this objective in mind, we define a methodology to analyse the competition among networks of any size or topology, demonstrating that several Nash equilibria can coexist, with some of them benefiting the strongest networks and others benefiting the weaker ones. Particularly, we report the existence of a wide regime of the system parameters in which every weak network can induce the rest to cooperate in order to escape from a detrimental Nash equilibrium, taking over the final situation of the whole network-of-networks. Paradoxically, the strong network cannot reverse this phenomenon. This counterintuitive asymmetry that promotes the cooperation among weak networks is independent of the network structure or the competition rules, and it could be applicable to an extensive number of real systems [15].

[1] M. E. J. Newman, *Networks: An Introduction* (Oxford University Press, New York, 2010).

[2] R. Pastor-Satorras and A. Vespignani, Epidemic spreading in scale-free networks, Phys. Rev. Lett. **86**, 3200-3203 (2001).

[3] D. Centola, The spread of behavior in an online social network experiment, Science **329**, 1194-1197 (2010).

[4] T. Gross and B. Blasius, Adaptive coevolutionary networks: a review, J. R. Soc. Interface **5**, 259-271 (2008).

[5] F. C. Santos, M. D. Santos, and J. M. Pacheco, Social diversity promotes the emergence of cooperative behavior, Nature **454**, 213-216 (2008).

[6] L. Dall'Asta, M. Marsili, and P. Pin, Collaboration in social networks, Proc. Natl. Acad. Sci. U.S.A. **109**, 4395-4400 (2012).

[7] Z. Wang, A. Szolnoki, and M. Perc, Self-organization towards optimally interdependent networks by means of coevolution, New J. Phys. **16**, 033041 (2014).

[8] Z. Wang, L. Wang, A. Szolnoki, and M. Perc, Evolutionary games on multilayer networks: a colloquium, Eur. Phys. J. B **88**, 124 (2015).

[9] M. Kivelä, A. Arenas, M. Barthelemy, J. P. Gleeson, Y. Moreno, and M. A. Porter, Multilayer networks, J. Complex Netw. **2**, 203-271 (2014).

[10] S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, and S. Havlin, Catastrophic cascade of failures in interdependent networks, Nature **464**, 1025-1028 (2010).

[11] F. Radichi and A. Arenas, Abrupt transition in the structural formation of interconnected networks, Nat. Phys. **9**, 717-720 (2013).

[12] J. Aguirre, R. Sevilla-Escoboza, R. Gutiérrez, D. Papo, and J. M. Buldú, Synchronization of interconnected networks: the role of connector nodes, Phys. Rev. Lett. **112**, 248701 (2014).

[13] J. Nash, Equilibrium points in n-person games, Proc. Natl. Acad. Sci. U.S.A. **36**, 48-49 (1950).

[14] J. Aguirre, D. Papo, and J. M. Buldú, Successful strategies for competing networks, Nat. Phys. **9**, 230-234 (2013).

[15] J. Iranzo, J. M. Buldú, and J. Aguirre, Competition among networks highlights the power of the weak, Nat. Commun. **7**, 13273 (2016).