Hydrodynamic quantization in bouncing droplets

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Over the past few years, a new set of experiments consisting of a milimetric droplet bouncing on an excited (below Faraday's threshold) liquid surface [1] has shown in a macroscopic system features that were assumed of only purely quantum systems, such as the interference pattern through a double slit [2], tunneling [3], quantization of periodic orbits [4], or orbital level spliting [5].

The fascinating analogies existing between quantum systems and bouncing droplets can be understood through the pilot-wave theory developed by Louis de Broglie [6]: In the hydrodynamic setting, the vibrating surface of the fluid plays the role of the underlying (quantum) pilot-wave, whose shape determines the dynamics of the particle (bouncing droplet).

In this communication [7], we push the analogies between bouncing droplets and quantum systems one step further by demonstrating the existence of some hydrodynamic constant in the limit of high memory, which plays a role similar to that of the Planck constant in quantum mechanics. For that purpose, we analyze the dynamics of a bouncing droplet in the highly chaotic quartic potential [8, 9]

$$V(x,y)=\frac{1}{2}x^2y^2+\frac{1}{400}(x^4+y^4), \qquad (1)$$

by solving the corresponding equation of motion [1]

$$\kappa\,\ddot{\mathbf{q}}(t)+\dot{\mathbf{q}}(t)=-\nabla V(\mathbf{q}(t))-\beta\,\nabla\psi(\mathbf{q}(t)), \qquad (2)$$

where $\mathbf{q}(t)$ is the particle position vector at time t, κ is the reduced mass of the particle, β is the memory strength parameter, and $\psi=\int_{-\infty}^{t} J_0(|\mathbf{q}-\mathbf{q}(s)|)\,e^{-(t-s)}\,ds$ is a term, evaluated along the droplet trajectory, accounting for the force exerted by the liquid surface.

Panel (a) of Fig. 1 shows the average height of the liquid surface, i.e., the mean field, as a function of the position x and β. The liquid surface shows a very clear regular pattern at certain values of the memory strength β, with a number of nodes that increases with the value of the parameter, as shown in panels (b)-(d).

The main conclusion of this Communication is that the position of these maxima satisfy a quantization condition similar to that of Bohr-Sommerfeld formulae for the wave functions of quantum mechanics.

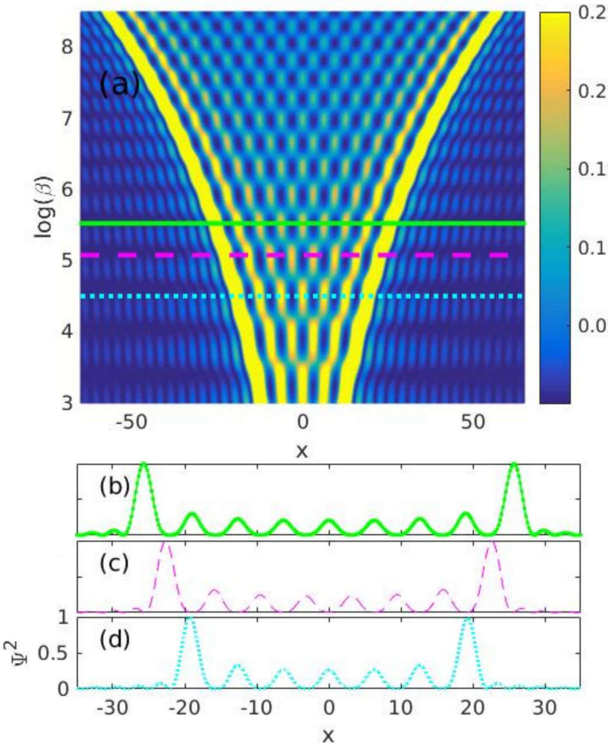

Fig. 1. Time-average for the square of the liquid wave function for a reduced mass $\kappa = 8$. (a) Surface as a function of the memory strength β and the position x. (b)-(d) Sections for a constant memory strength equal to (b) $\beta = 250$, (c) $\beta = 160$, and (d) $\beta = 90$.

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