Nonlinear population dynamics in a bounded habitat

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Population dynamics is constrained by the environment, which needs to obey certain conditions to support individual's survival. Through the last decades, theoretical developments have been made to identify the environment spatiotemporal structures for which the population is sustainable. These advances comes in crucial times where habitats have been suffering drastic transformations, from degradation of the landscape to climate changes.

In order to understand this issue, one typically looks for the critical line in the environment parameter space that separates the population survival and extinction phases. In the classical work by Skellam [1], a single population lives in a static habitat domain of size L. In this minimal case, simple laws for individual behavior are considered, assuming, for instance, random motion and density independent reproduction rates. In terms of the balance between the spatial scales present, it is straightforward to find that exists an L_c above which the population survives. In Fig. 1 we show the temporal evolution of the population density distribution for habitat size below, at and above L_c. Since then, many improvements have been made considering different habitat boundary conditions, individual behavior and time variability. Nevertheless, despite the fact that L_c is sensitive to the modeling details, it is a common feature that L_c is always a lower bound to habitat size.

In this work [2], we investigate an extension of this problem, proposing a general formulation that introduces density-dependent feedbacks in organisms' mobility and reproduction rate. Explicitly, we address the class of dynamics given by $\partial_t \rho = \partial_x(\rho^{\nu-1}\partial_x\rho) + \rho^{\mu} + \mathcal{O}(\rho^{\mu+\varepsilon})$, where μ and ν are real parameters that regulate the degree of nonlinearity present. We obtained exact expressions for the critical habitat size L_c and the profiles for the steady states. Together with numerical simulations, we show that depending on the region in the parameter space $\mu - \nu$, the population survival can occur for either for $L \geq L_c$, $L \leq L_c$ or even for any L (see Fig. 2). This generalizes the common statement that L_c represents the minimum habitat size. In addition, nonlinearities introduce dependence on the initial conditions, affecting L_c. We show that, despite the counterintuitive changes in the stability on the critical line L_c, as more individuals are introduced the survival phase is always increased.

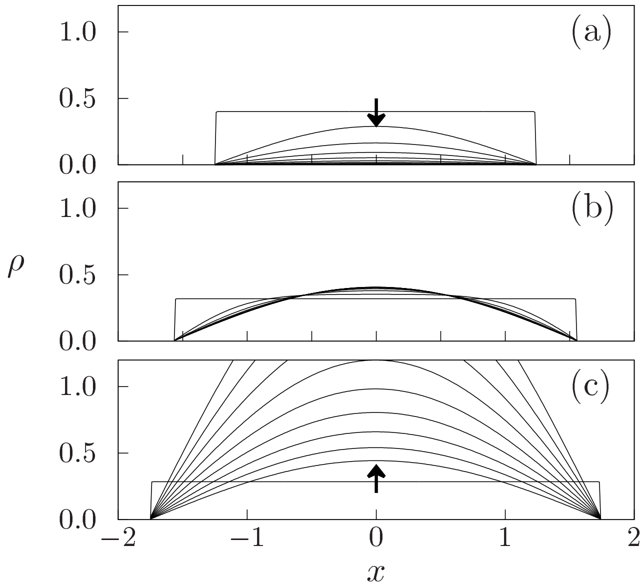

Fig. 1. Temporal evolution of the density distribution profile for the linear case in a one-dimensional habitat domain with harsh boundary condition. For (a) $L < L_c$, (b) $L = L_c$ and (c) $L > L_c$, the population becomes extinct, attains a steady state or blows up, respectively. The lines are produced by the analytical solutions. The arrows indicate the direction of time.

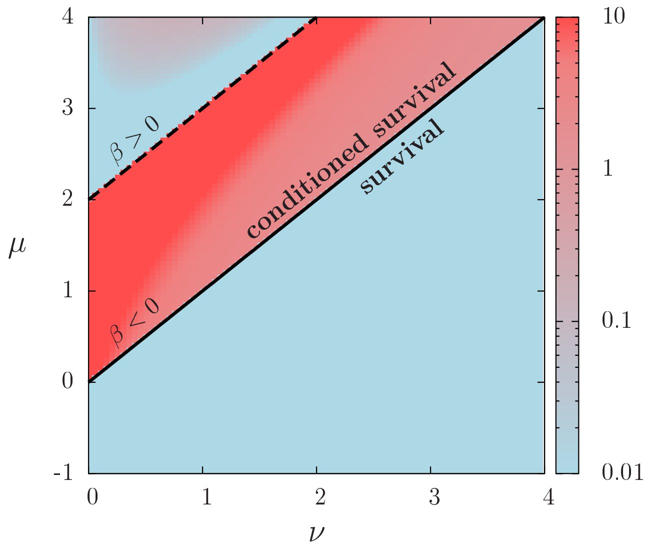

Fig. 2. Color map of the habitat size L_c in the plane (ν, μ), for initial condition $N_0 = 1$. The solid line $\mu = \nu$ separates the phases where survival always occurs (below) or it is conditioned (above). The dotted line separates where survival occur for $L > L_c$ (below) or $L < L_c$ (above). The parameter $\beta = 1 + 2/(\mu - \nu - 2)$, rules the scaling relation between $L_c \sim N_0^{\beta}$, where N_0 is the initially introduced population size. At the dotted line, $L_c \to \infty$.

[1] J. G.Skellam, Random dispersal in theoretical populations, Biometrika **38**, 196-218 (1951).

[2] E. H. Colombo and C. Anteneodo, Nonlinear population dynamics in a bounded habitat, J. Theor. Biol. **446**, 11-18 (2018).