Stochastic animal foraging models with resets: General approach and new ingredients

Axel Masó-Puigdellosas, Daniel Campos, and Vicenç Méndez
Grup de Física Estadística, Departament de Física, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

In the last decades a lot of effort has been put into the description of the territorial motion of animals [1]. Among the models that have been proposed, diffusion-based models, Lévy flights and Lévy walks are three of the most used. Nevertheless, in the vast majority of these approaches, only the foraging process of the animals was described (i.e., their motion while they are collecting) and it had not been attempted to study their overall behaviour, including the return to the nest after their search is completed.

On this direction, in 2011 Evans and Majumdar [2] studied the properties of a model consisting on a diffusive process subject to resets with constant rate (i.e., the reset times are exponentially distributed). For this process, the mean first passage time (MFPT) is finite and the mean square displacement (MSD) collapses to a stationary value unlike a simple diffusive process which has infinite MFPT and the MSD scales linearly with time. The fact that the system achieves a stationary state allows us to define the territoriality of a given species being a quantitative measure of the region that the animals occupy around its nest.

From then on, multiple processes have been studied when they are subject to resets but no general study of the robustness of the properties found in [2] has been done. In this work we address this issue by analysing general properties of foraging processes with resets from a continuous-time random walk (CTRW) perspective. Moreover, we use a Mittag-Leffler distribution for the reset times, a generalisation of the exponential distribution which can also take the form of a long-tailed distribution.

On one hand, we derive a general equation for the MSD of the process with resets in terms of the MSD of the intrinsic process. From it, we conclude that all the processes whose MSD is Laplace-transformable and finite reach a stationary value when exponentially distributed resets are applied to them. On the other hand, we study the finiteness of the mean first arrival time (MFAT) of the process. We derive the conditions under which it is finite for a general power-law asymptotic behaviour of the intrinsic survival probability $Q_x(t) \propto t^{-q}$ and study some cases of particular interest as the diffusive process or the Lévy flights.

With the aim of adjusting the present formulation to real cases, we also propose a new model for the description of the movement of the animals consisting in the introduction of a new element to the CTRW with resets: a retention mechanism at the origin. This novel ingredient is motivated by the fact that when animals return to their nest after a collecting trip, they rest there some time before starting another trip. For this model we also study the MSD and the MFAT for general intrinsic processes and reset and retention time distributions, studying more deeply some cases of particular interest in ecology.

[1] F. Bartumeus, M. G. E. Da Luz, G. M. Viswanathan, and J. Catalan, Animal search strategies: a quantitative random-walk analysis, Ecology **86**, 3078-3087 (2005).

[2] M. R. Evans and S. N. Majumdar, Diffusion with stochastic resetting, Phys. Rev. Lett. **106**, 160601 (2011).