Quantum approach to opinion dynamics

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Binary-state models have been widely used to study the emergence of collective phenomena. In particular, Voter Model (VM) [1, 2] is a binary-state model based on local interactions. In VM, just as in an Ising model, nodes can have two states, that we can call spin up (1) or spin down (-1). In a single event, a randomly chosen node copies the state of one of its neighbours, also chosen at random. The Voter model has an analogue in quantum mechanics, the Heisenberg Model that is the quantum version of the classical Ising model. As it is well known, at $T = 0$ the Ising Model reduces exactly to the Voter Model in one dimension, while for higher dimensions the main difference is the absence of surface tension at the domain boundaries in the Voter Model.

We would like to define a "quantum-like" version of the pure VM. In the classical version, the key feature is the imitation dynamics: When an individual evolves, it simply copies the state of a neighbor. Therefore, let us consider a system where the only quantum entities are the opinions inside individuals' minds: An opinion really exists only when the agent has to express it based on its actual mental state. On the other hand, such mental state is the generic set of the thoughts, beliefs, views owned by the individual, which can be communicated and shared interacting with the others. In fact, we consider the state of mind $|\psi_i\rangle$ as a quantum object regarding internal dynamics, but treat it classically for what concerns the external dynamics. This would not make any sense for a real quantum system, yet for this very reason we call this model "pseudo-quantum" Voter Model (PQVM).

The Model – we consider a system of N agents whose state of mind is determined by their state ket $|\psi_i\rangle$ ($i \in \{1, N\}$). Since we are dealing with a two-opinion problem, such states belong to the Hilbert space of the $\frac{1}{2}$spin systems (spinors). Let $|u\rangle$ and $|d\rangle$ be the base kets, representing the positive and negative opinions, respectively, along a certain direction z (which can be seen as a particular binary question, i.e., yes or no in a referendum, the choice between two candidates, etc.), so that

$$|\psi_i\rangle = \alpha_i|u\rangle + \beta_i|d\rangle, \tag{1}$$

with $\alpha_i^*\alpha_i + \beta_i^*\beta_i = 1$. The other directions represent different questions, decreasingly related to the z-axis one as the angle increases. By constructions, the x and y directions, in a three dimensional space, should represent opinions that are completely independent from the studied one.

The dynamics takes place analogously to the classical Voter Model: At each elementary time step an agent is picked up at random and imitates the state of one of its neighbours, also randomly selected. The simplest assumption is independence: in the basic model, the states of different individuals cannot be directly correlated (unless one has just imitated a neighbor), so that the global state of the system will be a product-state

$$|\Psi\rangle = \Pi_i|\psi_i\rangle. \tag{2}$$

We expect the phenomenology of the PQVM to be much richer than the VM, and depends even more heavily on initial conditions.

Preliminary results – Analogously to the VM, the final state of the system is necessarily the one where all the individuals share the same state

$$|\psi_i^{\text{fin}}\rangle = |\phi^\infty\rangle, \ \forall i. \tag{3}$$

By definition of the model, that works by copying neighbours' states, $|\phi^\infty\rangle$ is necessarily one of the initial states which finally survived to the dynamics and the frozen configuration is reached following a power law of time with exponent $1/2$, exactly as in the CVM, even though with infinitely larger number of available states. If we are interested only in the final state, forcing a little bit the formalism, we can write down an Hamiltonian whose ground level (which is infinitely degenerate) is exactly the frozen state of the PQVM

$$\hat{H} = -J \sum_{\langle i,j\rangle} \|\langle\psi_j|\psi_i\rangle\|^2 \, \hat{I}, \tag{4}$$

where $\hat{I}$ is the identity operator and J the coupling constant, which can be assumed equal to 1. Similarly to the classical Ising Model, the system described by this Hamiltonian at zero temperature is perfectly equivalent to the PQVM in one dimension, differing for the absence of surface tension in higher dimensions [3].

As confirmed by simulations, differently from the classical version, the fact that the system finally reaches uniformity (i.e., all the agents are in the same state) does not imply that when they express their opinions, the population will be in a consensus state: since expressing an opinion on a given topic is equivalent to accomplishing a measurement of the spin along a given direction, in general every single opinion will depend in a probabilistic way from the mental state of the agent which expresses it. Moreover, the actual opinion distribution will also depend heavily from the initial conditions. Deeper and more systematic studies will be soon accomplished.

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