

## Structure *versus* dynamics in neuronal complex networks

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According to Graph Theory, complex networks can be defined as a set of nodes (or vertices),  $V = \{1, 2, \dots, N\}$ , and a set of edges (or links),  $L$ , that describe the connections between nodes [1]. In our case, the nodes will be the neurons of our culture; whereas the links will be their synaptic connections. These networks are represented with a matrix, called the adjacency matrix,  $A$ , which, in the simplest instance, will be binary. If the  $i$ -th element is connected to the  $j$ -th, then  $A_{ij} = 1$ ; otherwise,  $A_{ij} = 0$ .

The networks are created in a way that emulates the growth of a biological culture of neurons over a surface. A generalization of the method put forward by Orlandi *et al.* [2] will be used, where a connection between neurons will happen only if two conditions are met. First, the axon of a given neuron intersects the dendritic tree of any other neuron. And second, those neurons that fulfill the first condition will connect with probability  $\alpha$ , which is independent of the overlapping length between the axon and the dendritic tree that is intersected. Thus, obtaining neurons such as the one presented in Fig. 1 and networks as in Fig. 2.

To describe the dynamics we will use the Izhikevich model [3],

$$\tau_c \dot{v} = k(v - v_r)(v - v_t) - u + I + \eta, \quad (1a)$$

$$\tau_a \dot{u} = b(v - v_r) - u, \quad (1b)$$

which depicts the neuron through two variables: the membrane potential  $v$ , and the inhibitory current  $u$ . It also introduces the influence of synaptic currents through the term  $I$  in Eq. (1a).

Now that both the structural and dynamical models have been explained, in order to characterize our cultures we carry out a series of structural and dynamical measurements. This way we will try to describe the percolation transition and the transition to synchronization when varying the connection probability  $\alpha$ , as well as the resistance to failures of our cultures.

To study de percolation transition we measure the average connectivity and the size of the giant component of our cultures. We see that both of these measurements grow rapidly till they saturate. However, the size of the giant component grows significantly faster than de average connectivity. This means that, as  $\alpha$  increases, the skeleton of network will be completely formed before it has made all possible connections.

To describe the transition to synchronization, three parameters will be used. A microscopic parameter, designed to account for the pairs of neurons that are synchronized; a macroscopic parameter to show when most of the neurons of our culture are synchronized; and the size of the neuronal avalanches, as defined by Beggs and Plenz in [4]. The results we got from these measurements show that there exists a microscopic synchronization regime in absence of

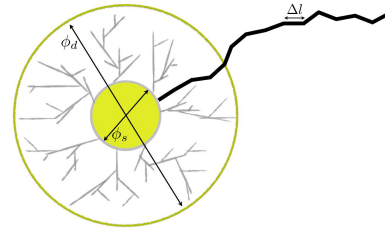


Fig. 1. Model of a neuron:  $\phi_s$  is the diameter of the soma,  $\phi_d$  is the diameter of the dendritic tree, and the axon is built concatenating segments of length  $\Delta l$ .

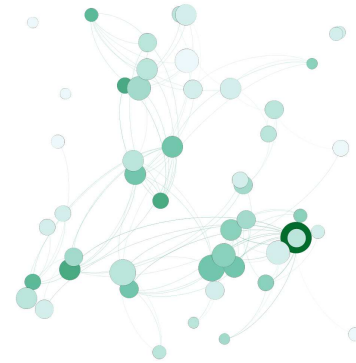


Fig. 2. Example of a network obtained from a culture with 50 neurons, a density of 10 neurons  $\text{mm}^{-2}$ , and  $\alpha = 0.75$ .

macroscopic synchronization. This behavior has also been observed in Kuramoto oscillators [5].

Finally, to test the resistance of our cultures to failures, we counted the number of avalanches that took place after we had removed the neurons that initiated most of them and compared it to the number of avalanches that happened before removing any neurons. Here, we found out that so long as we do not remove all the neurons that initiate the avalanches, the culture will remain just as active .

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