

# Granular gas mixtures of inelastic rough particles: Hard disks and hard spheres

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Granular gas mixtures modeled as systems of inelastic and rough particles, either hard disks on a plane or hard spheres, are considered. Both classes of systems are embedded in a three-dimensional space ( $d = 3$ ) but, while in the hard-sphere (HS) case the translational and angular velocities are vectors with the same dimensionality  $d_t = d_r = 3$ , in the hard-disk (HD) case the translational velocity vectors are planar (i.e.,  $d_t = 2$ ) and the angular velocity vectors are orthogonal to the motion plane (i.e.,  $d_r = 1$ ). This complicates a unified presentation of both classes of systems, in contrast to what happens for smooth, spinless particles, where an unambiguous kinetic-theory treatment of  $d$ -dimensional spheres is possible [1].

The kinetic-theory derivation of the energy collisional production rates  $\xi_{ij}^{\text{tr}}$  and  $\xi_{ij}^{\text{rot}}$  (where the indices  $i$  and  $j$  label different components) has been separately carried out for HS [2] and HD [3] multicomponent granular gases. The major aim of this work is to unify those studies by expressing  $\xi_{ij}^{\text{tr}}$  and  $\xi_{ij}^{\text{rot}}$  in terms of the dimensionality  $d_t$ , after setting  $d_r = 2d_t - 3$ . The HS and HD expressions are recovered by particularizing to  $d_t = 3$  and  $d_t = 2$ , respectively. Moreover, in the case of spinless particles with  $d = d_t$ , known energy production rates  $\xi_{ij}^{\text{tr}} = \xi_{ij}$  of smooth  $d$ -dimensional spheres [1] are recovered.

Our results are applied to a comparative analysis of the homogeneous free cooling of HD and HS gases. As an illustration, Fig. 1 shows a density plot of the rotational/translational temperature ratio as a function of the coefficients of normal ( $\alpha$ ) and tangential ( $\beta$ ) restitution. As can be observed, the disparity between both types of temperature is generally more pronounced in the case of disks than in the case of spheres. A similar behavior is exhibited by the rotational/translational nonequipartition in binary mixtures; however, the component/component degree of nonequipartition is stronger in HS gases than in HD gases.

A.M. is grateful to the *Ministerio de Educación, Cultura y Deporte* (Spain) for a *Beca-Colaboración* during the academic year 2017-2018. The research of A.S. has been supported by the *Ministerio de Economía y Competitividad* (Spain) through Grant No. FIS2016-76359-P and by *Junta de Extremadura* (Spain) through Grant No. GR18079, both partially financed by *Fondo Europeo de Desarrollo Regional* funds.

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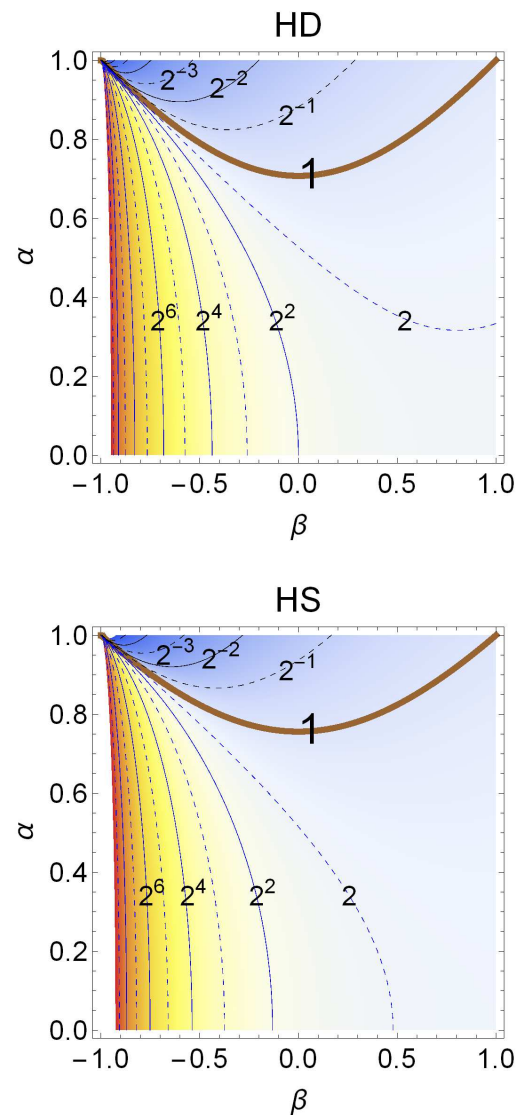


Fig. 1. Density plot of the rotational/translational temperature ratio for HD (top) and HS (bottom) granular gases. The contour lines correspond to the values 1 (thick solid lines),  $(2^{-1}, 2^{-2}, \dots)$ , and  $(2, 2^2, \dots)$

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